

**USA CAS<sup>9</sup>, Ohio 2015 – Kilts and CAS in Statistics**

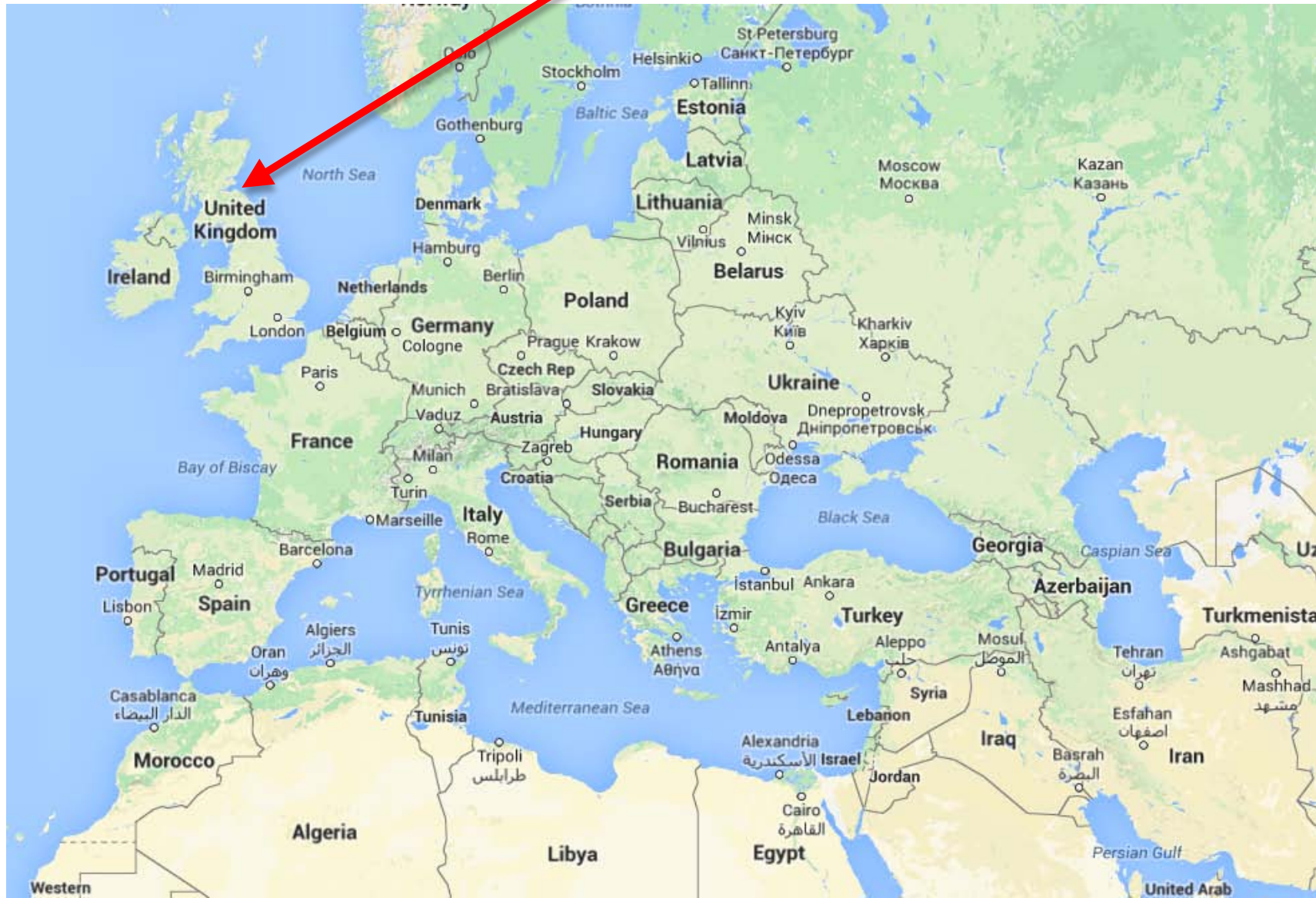
# Statistics

**Nevil Hopley**

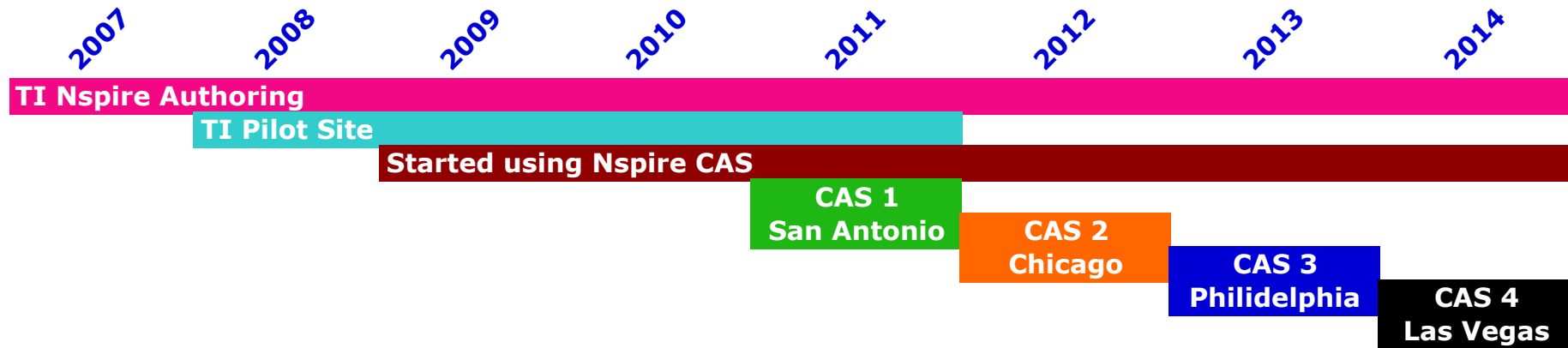
**T<sup>3</sup> National Trainer, Scotland & UK.  
Head of Mathematics Department**

**[www.calculatorsoftware.co.uk/nspire](http://www.calculatorsoftware.co.uk/nspire)**

# My Home



# My CAS Timeline



## CAS Talks at TI International Conferences

- 2011 My first 18 months of CAS usage
- 2012 Trigonometry and Rearranging Equations
- 2013 Linear Equations and Units
- 2014 Extending CAS with functions and programs
- 2015 CAS in Statistics (v1) at T<sup>3</sup> Europe, Madrid



## Credits

Chris Harrow, Ohio, USA

John Hanna, Honolulu, USA

Pat Mara, Pueblo, USA

[www.statlect.com/distri.htm](http://www.statlect.com/distri.htm)

[answers.yahoo.com/question/index?qid=20120403201108AAIaU1o](http://answers.yahoo.com/question/index?qid=20120403201108AAIaU1o)

[math.stackexchange.com/questions/117926/finding-mode-in-binomial-distribution](http://math.stackexchange.com/questions/117926/finding-mode-in-binomial-distribution)

# Chris Harrow



[casmusings.wordpress.com](http://casmusings.wordpress.com)

expand $\left((heads+tails)^6\right)$

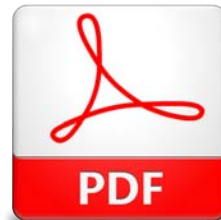
$heads^6 + 6 \cdot heads^5 \cdot tails + 15 \cdot heads^4 \cdot tails^2 + 20 \cdot heads^3 \cdot tails^3 + 15 \cdot heads^2 \cdot tails^4 + 6 \cdot heads \cdot tails^5 + tails^6$

# John Hanna



“LinReg Exposed!”

A talk from 2014 TI International Conference

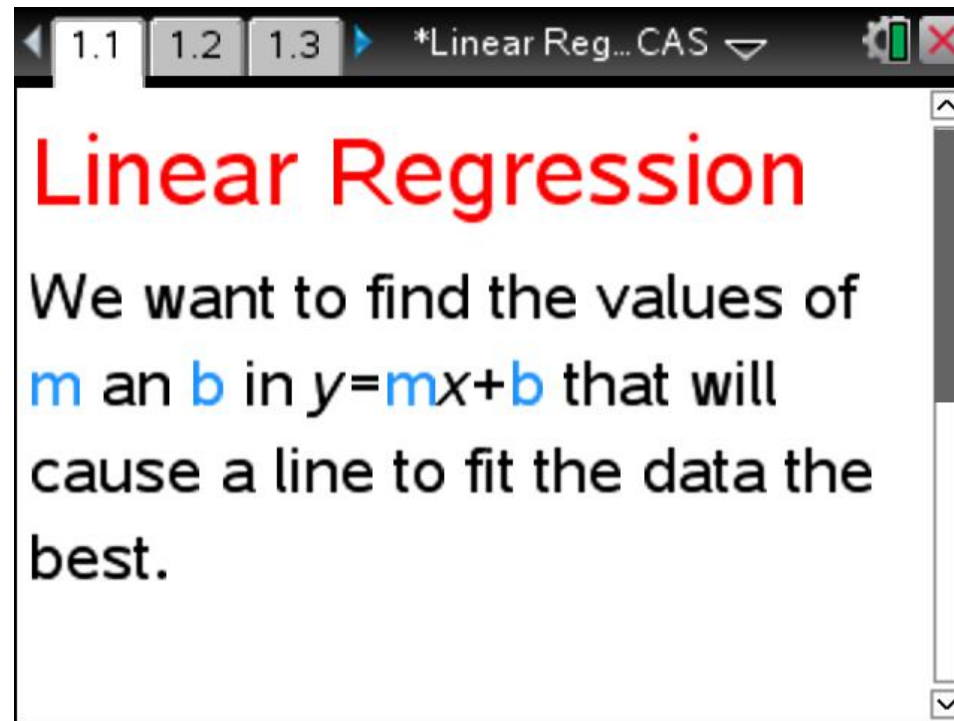


# Pat Mara



Linear Regression CAS.tns  
(Nspire Google Group, 19 Nov 2014)

**Linear Regression CAS - Pat Mara.tns**





# Today

Formulae for Standard Deviation

Discrete Uniform Distribution

Geometric Distribution

Laws of Expectation and Variance

Binomial Distribution

Poisson Distribution

Probability Generating Functions

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# Formulae for Standard Deviation

$$s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1} \quad \text{or} \quad s^2 = \frac{\sum_{i=1}^n x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1}$$

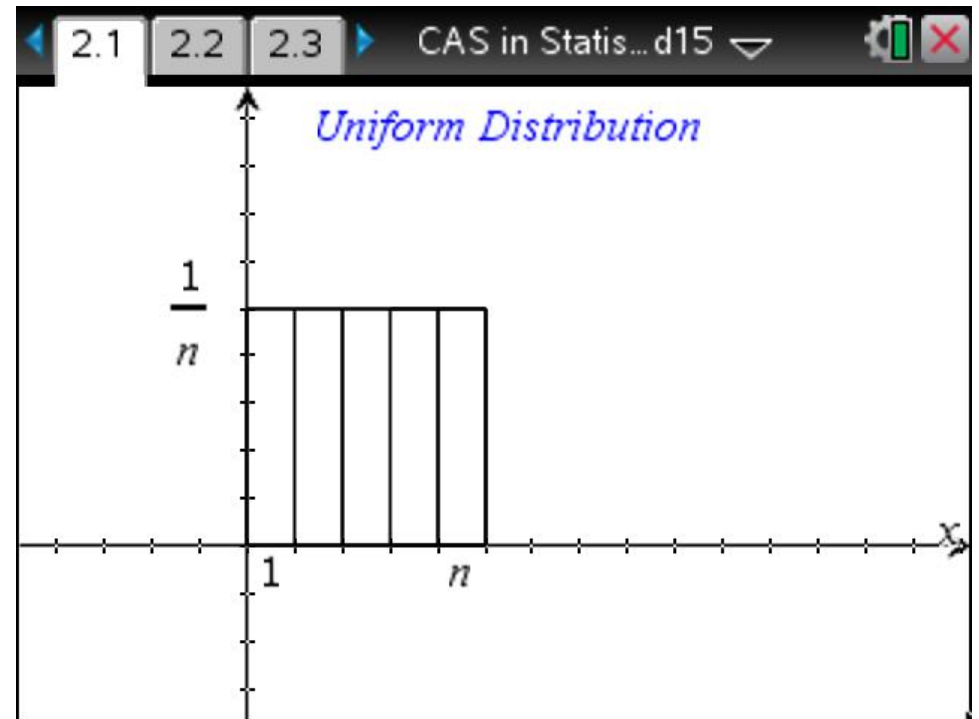
$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n} \quad \text{or} \quad \sigma^2 = \frac{\sum_{i=1}^n x_i^2 - \frac{(\sum x_i)^2}{n}}{n}$$

**Kilts and CAS in Statistics.tns pages 1.1 & 1.2**

# Discrete Uniform Distribution $\sim U[1,n]$

$$\begin{aligned} E(X) &= \frac{\frac{1}{2}n(n+1)}{n} \\ &= \frac{1}{2}(n+1) \end{aligned}$$

$$\begin{aligned} \text{Var}(X) &= \frac{\frac{1}{12}n(n-1)(n+1)}{n} \\ &= \frac{1}{12}(n^2-1) \end{aligned}$$



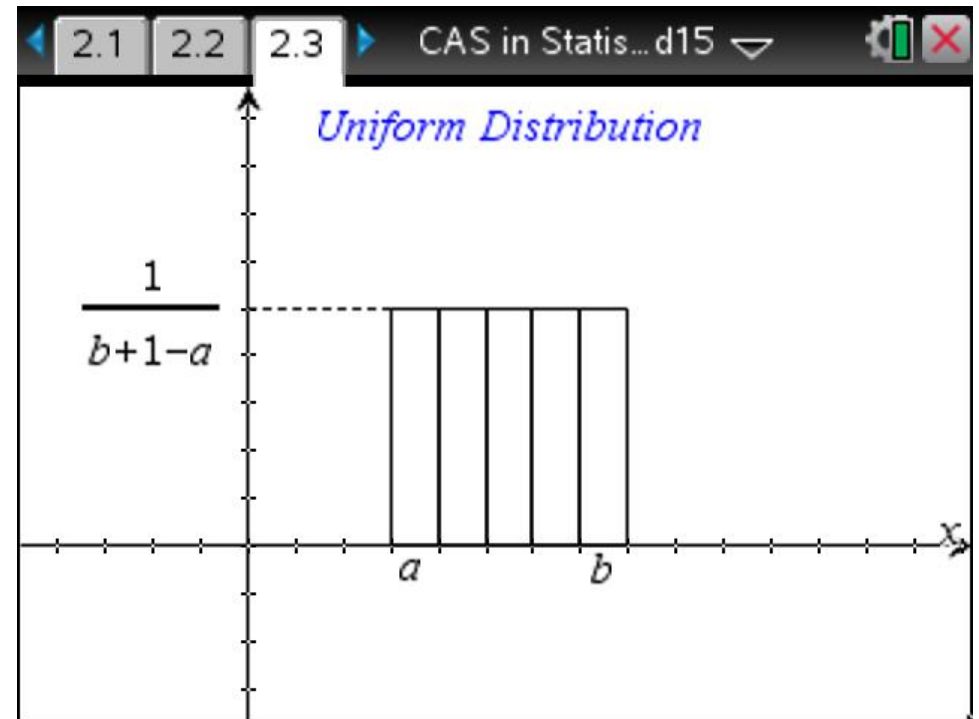
pages 2.1 & 2.2

# Discrete Uniform Distribution $\sim U[a,b]$

$$X \sim U[a,b]$$

$$E(X) = \frac{1}{2}(a+b)$$

$$Var(X) = \frac{1}{12}[(b-a+1)^2 - 1]$$



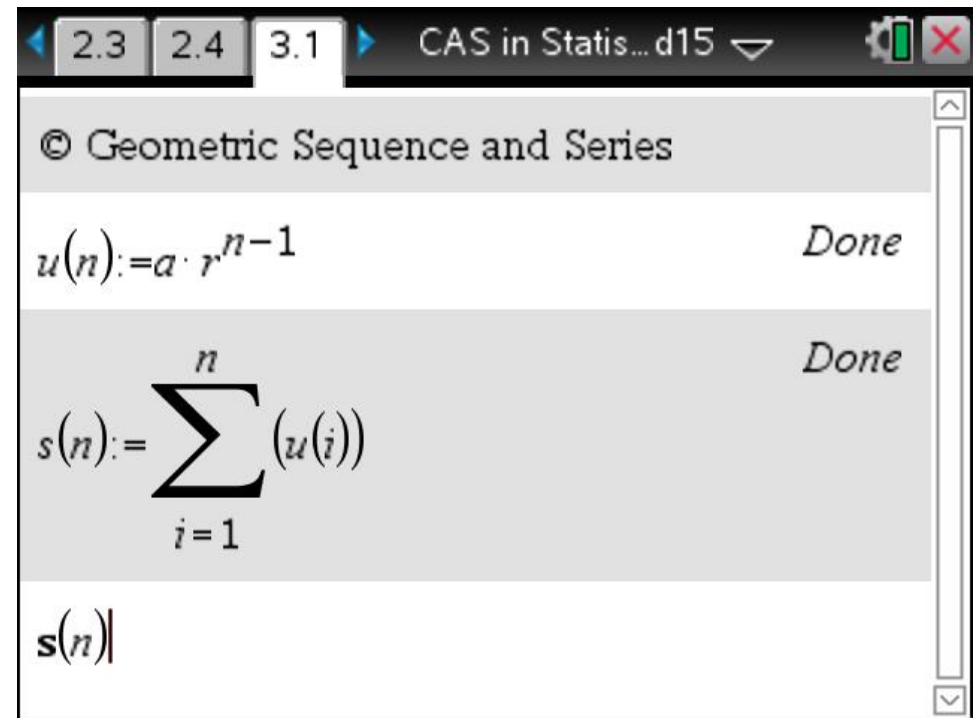
pages 2.3 & 2.4

# Geometric Sequences & Series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r}$$



**pages 3.1 & 3.2 & 3.3**

# Geometric Distribution

$X$  = number of trials until the first success

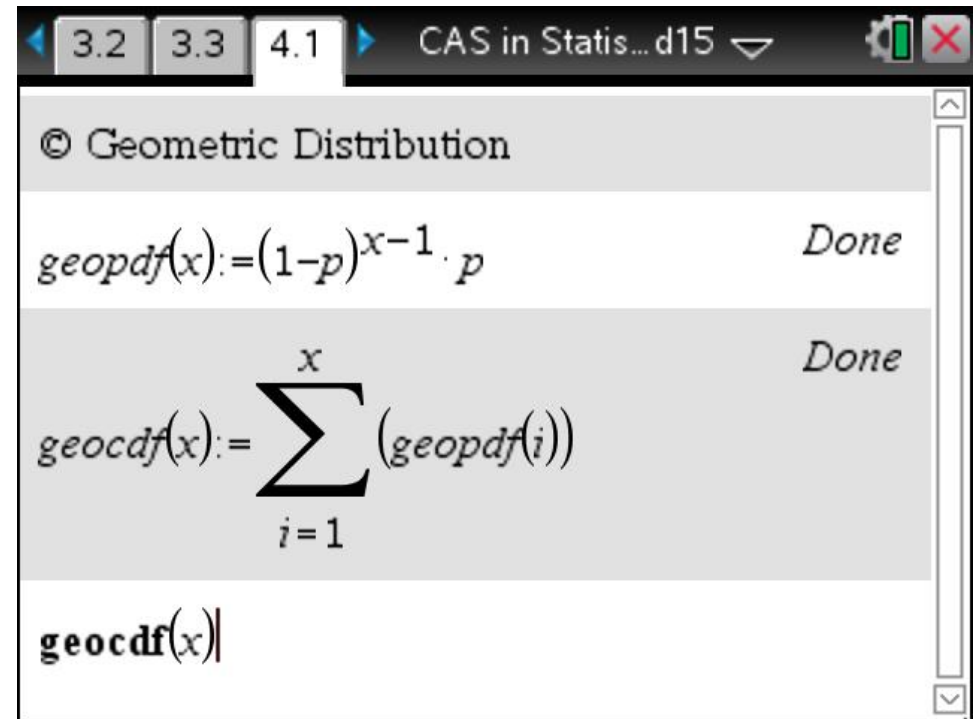
$P(\text{success on each trial}) = p$

$$P(X = x) = q^{x-1} p$$

$$P(X \leq x) = 1 - q^x$$

$$E(X) = \frac{1}{p}$$

$$\text{Var}(X) = \frac{q}{p}$$

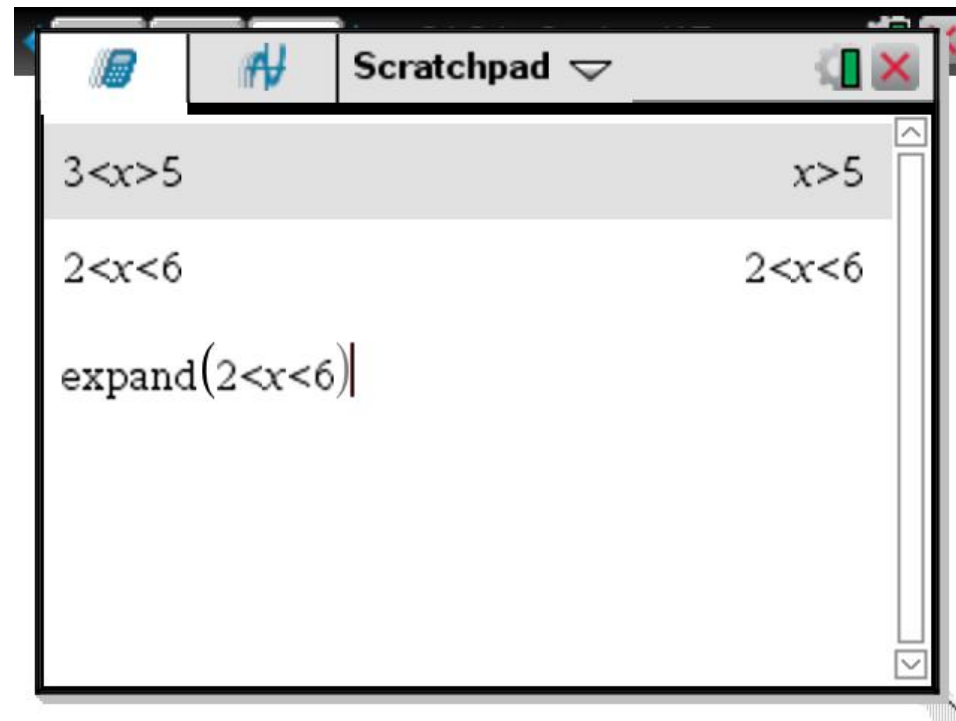


The screenshot shows a CAS window titled "CAS in Statis...d15" with tabs for pages 3.2, 3.3, and 4.1. The window content is titled "© Geometric Distribution" and contains the following definitions:

```
geopdf(x):=(1-p)x-1·p Done
geocdf(x):=∑i=1x(geopdf(i)) Done
geocdf(x)|
```

**pages 4.1 & 4.2 & 4.3**

# BONUS!



A screenshot of a Scratchpad window. The window title bar shows a keyboard icon, a handwriting icon, and the text "Scratchpad" with a dropdown arrow. The window contains the following text:

```
3<x>5          x>5
2<x<6          2<x<6
expand(2<x<6)|
```

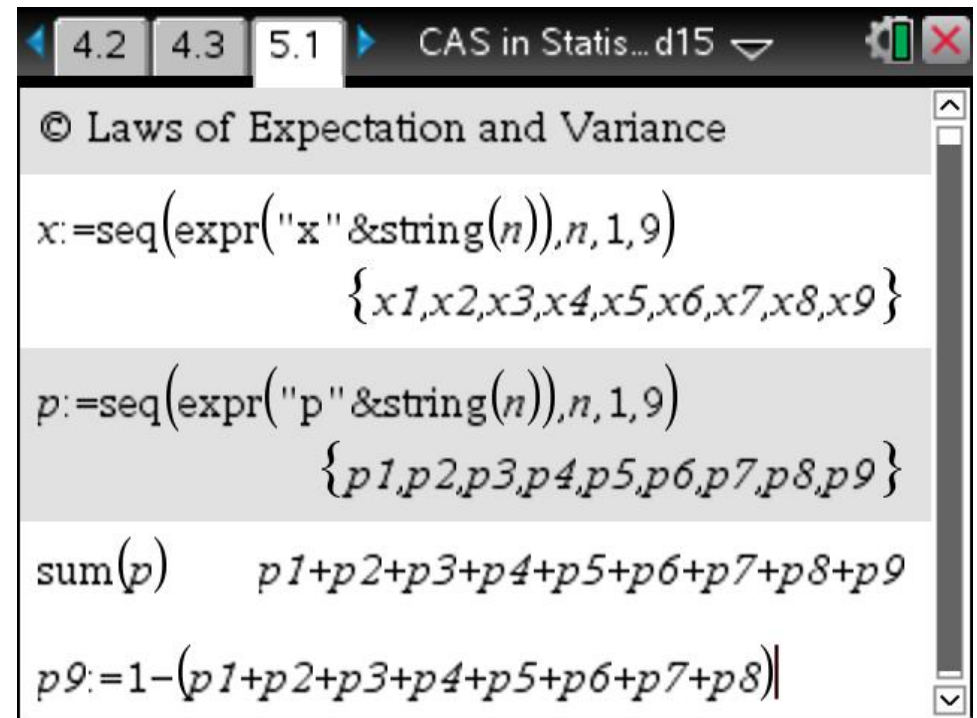
**Scratchpad**



# Laws of Expectation and Variance

$$E(aX + b) = aE(X) + b$$

$$\text{Var}(aX + b) = a^2 \text{Var}(X)$$



```
4.2 4.3 5.1 CAS in Statis...d15  
© Laws of Expectation and Variance  
x:=seq(expr("x"&string(n)),n,1,9)  
      {x1,x2,x3,x4,x5,x6,x7,x8,x9}  
p:=seq(expr("p"&string(n)),n,1,9)  
      {p1,p2,p3,p4,p5,p6,p7,p8,p9}  
sum(p)  p1+p2+p3+p4+p5+p6+p7+p8+p9  
p9:=1-(p1+p2+p3+p4+p5+p6+p7+p8)
```

**pages 5.1 & 5.2**

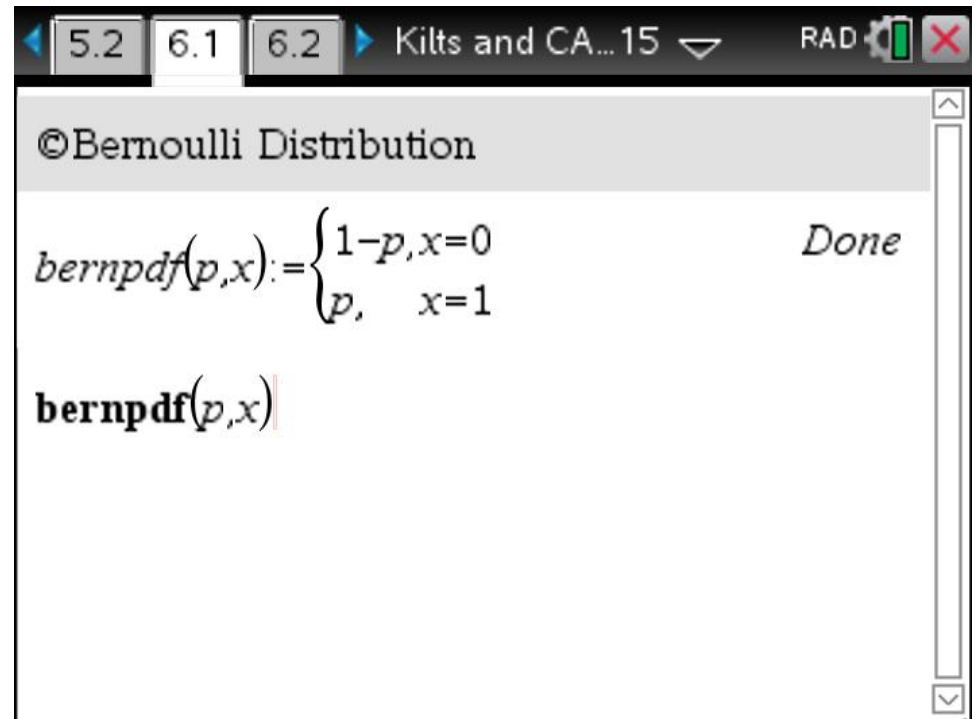
# Bernoulli Distribution

$X$  = number of successes in 1 trial  
 $P(\text{success on each trial}) = p$

$$P(X = x) = \begin{cases} 1 - p & \text{if } x = 0 \\ p & \text{if } x = 1 \end{cases}$$

$$E(X) = p$$

$$\text{Var}(X) = pq$$



```
5.2 6.1 6.2 Kilts and CA...15 RAD  
©Bernoulli Distribution  
bernpdf(p,x) := { 1-p, x=0  
                 p,   x=1  
                 Done  
bernpdf(p,x)
```

**pages 6.1 & 6.2 & 6.3**

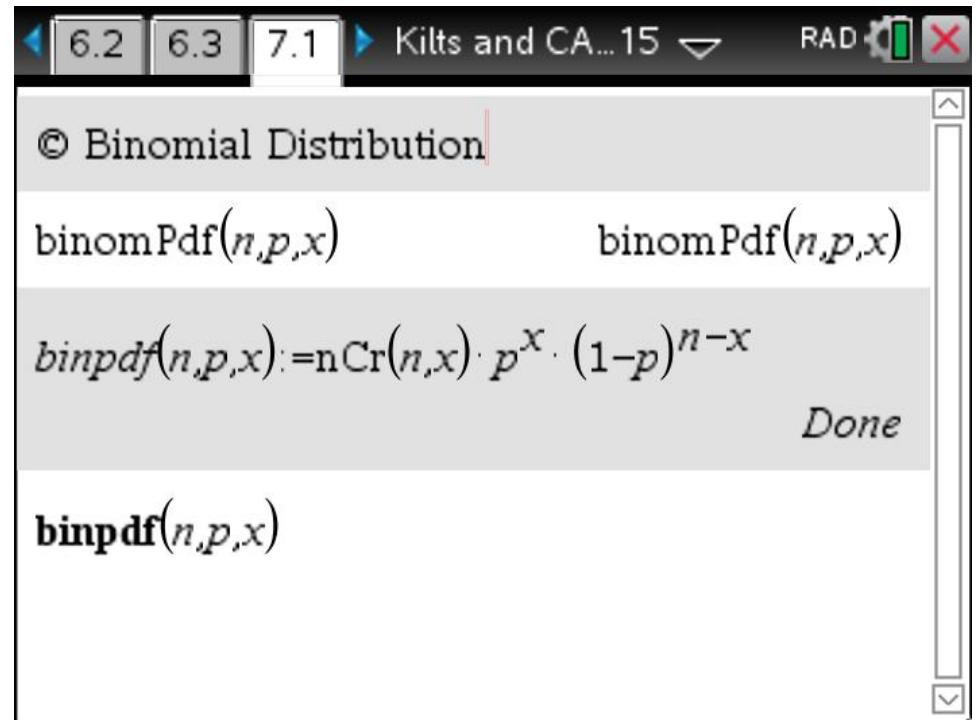
# Binomial Distribution

$X$  = number of successes in  $n$  trial  
 $P(\text{success on each trial}) = p$

$$P(X = x) = {}^n C_r p^x (1 - p)^{n-x}$$

$$E(X) = np$$

$$\text{Var}(X) = npq$$



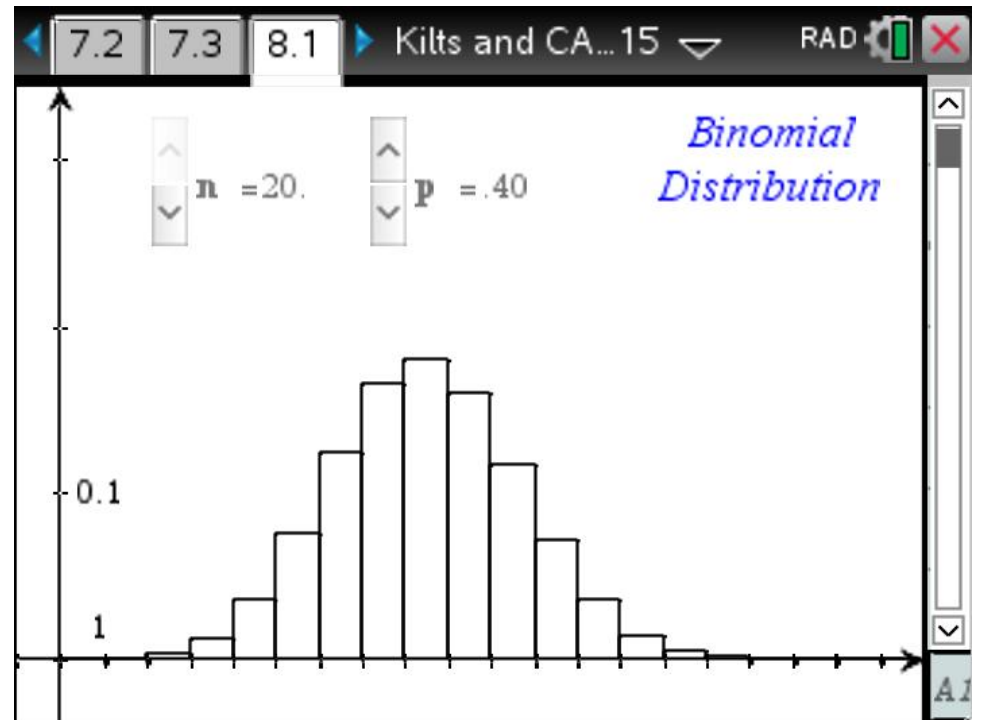
**pages 7.1 & 7.2 & 7.3**

# Mode of Binomial Distribution

*Mode is reached when:*

$$P(X = x + 1) \leq P(X = x)$$

$$\frac{P(X = x + 1)}{P(X = x)} \leq 1$$



**pages 8.1 & 9.1**

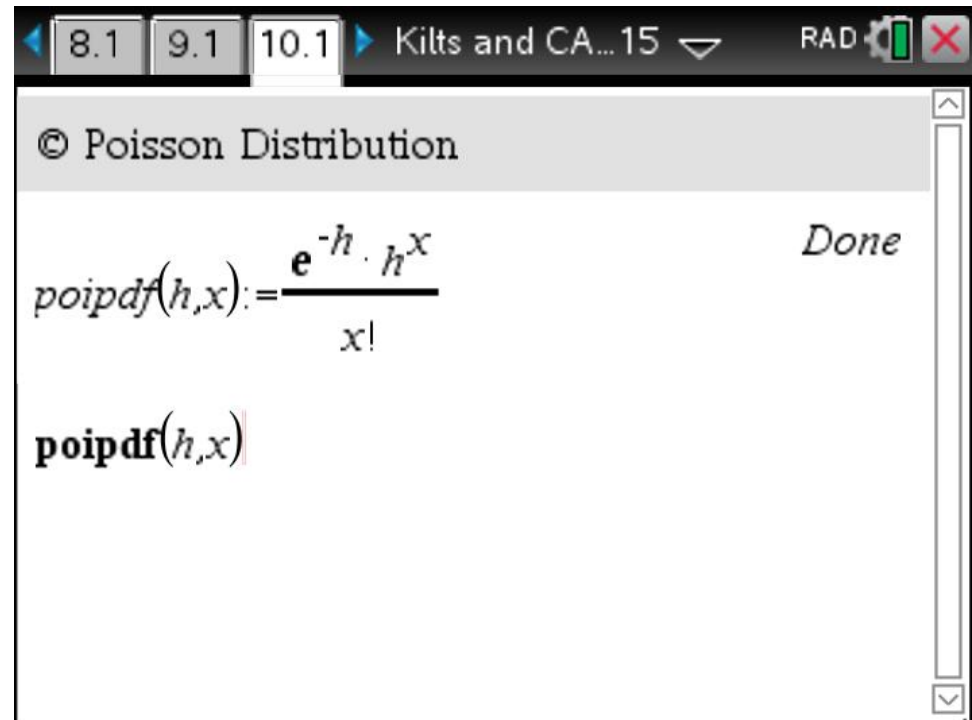
# Poisson Distribution

$X$  = number of events in a fixed interval  
mean rate of events =  $\lambda$

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$E(X) = \lambda$$

$$Var(X) = \lambda$$



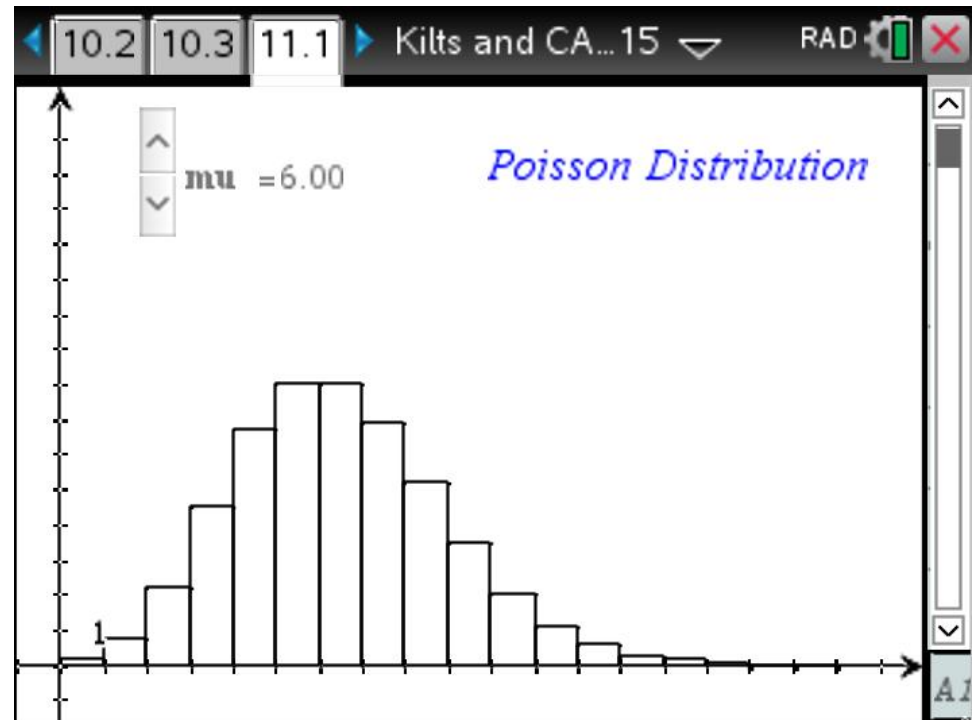
**pages 10.1 & 10.2 & 10.3**

# Mode of Poisson Distribution

*Mode is reached when:*

$$P(X = x + 1) \leq P(X = x)$$

$$\frac{P(X = x + 1)}{P(X = x)} \leq 1$$



**pages 11.1 & 12.1**

# Probability Generating Functions

$$G_X(t) = P(X = 0)t^0 + P(X = 1)t^1 + P(X = 2)t^2 + \dots$$

$$G_X(t) = \sum_{x=0}^n P(X = x)t^x$$

*A polynomial in  $t$  whose coefficients  
are the probabilities of the powers*

**pages 13.1 to 17.2**



$E(X)$  and  $\text{Var}(X)$  for both  $B(n,p)$  and  $\text{Poi}(\lambda)$   
Unbiased & Consistent Estimators  
Chi-Squared Distributions



**Want Copies of Everything?**

[www.calculatorsoftware.co.uk/nspire](http://www.calculatorsoftware.co.uk/nspire)

**Thank you for coming to my talk.**

**Nevil Hopley**

T<sup>3</sup> National Trainer, Scotland & UK.

Head of Mathematics Department

CAS user on Handhelds and TI-Nspire iPad App

TI-Basic and Lua Programmer

Mountain Unicycler